CONCEPTS' INTERRELATIONS

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SUMMARY

Concepts are couples of sets O and A. (O, A) gives the concept, that is the assignment, of the object 0 (a set of one or more elements -there is no real difference) to the set A of (common) attributes. The objects do not really exist: they just change according to the sequence of attributes. The objects are not definite, not standard. So, manipulations on objects are useless and meaningless. Besides, we realize, that a concept is not only an assignment from 0 to A but also an assignment from A to 0, that is dynamic. Exactly as multimedia are. The connections and links we need in multimedia are expressed, naturally, by concepts, since concepts are proved to have the structure of a lattice. So, we have a more complex order than linear and hierarchical ones. The lattice can be created by two algebraic operations. The operations we introduce, create a more rich lattice with more possibilities, combinations, varieties (different operations may create different lattices). So, our lattice serves as the deep knowledge for decision making, virtual reality and multimedia.

ΔΙΑΠΛΕΚΟΜΕΝΕΣ ΕΝΝΟΙΕΣ

Μεγ. Θ. Σωτηρόπουλος

ΠΕΡΙΛΗΨΗ

Έννοια ορίζουμε το ζεύγος των αντικειμένων αφενός και αφετέρου των κοινών τους ιδιοτήτων. Τα αντικείμενα αλλάζουν ανάλογα με τις ιδιότητες που εξετάζουμε. Άρα εκείνο που έχει σημασία είναι η σύνδεση μεταξύ των αντικειμένων και των ιδιοτήτων. Π.χ., όταν ο άνθρωπος δείχνει με το δάχτυλό του κάτι τότε δημιουργείται μία έννοια (δάχτυλο, δεικνυόμενο), ενώ τα ζώα δεν αντιλαμβάνονται αυτή τη σύνδεση.

Ανάμεσα στις έννοιες ορίζουμε πράξεις ένωσης, τομής, διαφοράς και συμπληρωματικής έννοιας. Παρατηρούμε τότε ότι από δύο έννοιες προκύπτουν άλλες τρεις, μία υπερκείμενη, μία υποκείμενη και μία ανέντακτη (μη διατεταγμένη). Άρα, η δομή των εννοιών που έχουμε δεν είναι ούτε απλά γραμμική ούτε δενδρική (ιεραρχική). Αποδεικνύεται ότι είναι η μαθηματική δομή του Συνδέσμου.

Συνεπώς, η ίδια η φύση, η ζωή και η γλώσσα μας οδηγούν σε διαπλεκόμενες έννοιες και όχι a priori ιεραρχίες. Θα δοθούν πολλά παραδείγματα από τη Γεωμετρία, τη Φυσική, τη Βιολογία, τη Γλωσσολογία, την Ιατρική κλπ.

1 Mathematical structure of concepts

Definition 1. Concept is every assignment of a prototype to an icon, whatever may be the prototype and the icon. We call the prototype "object" and the icon "attributes". We symbolize a concept with a couple whose left part is the object and right part the attributes.

Definition 2. $(O_1, A_1) \dot{\cup} (O_2, A_2) = (O_1 \cup O_2, A_1 \cap A_2)$, where $\cup$ and $\cap$ are the usual operations between sets, union and intersection, respectively.

Definition 3. $(O_1, A_1) \underset{\bullet}{\cap} (O_2, A_2) = (O_1 \cap O_2, A_1 \cup A_2)$,

With the above two operations, for every two concepts, there exist a "higher" and a "lower" concept.

Definition 4. $(O_1, A_1) \subseteq^{\bullet} (O_2, A_2) \Leftrightarrow (O_1 \subseteq O_2$ and $A_1 \supseteq A_2)$, where $\subseteq$ and $\supseteq$ denote the usual subset and superset, respectively. The subordinated concept (O_1, A_1) are the species and the superordinated concept (O_2, A_2) is the genus.

Definition 5. The complement of the concept (O, A) is the concept (O^C, A^C), where O^C and A^C are the usual set-theoretic complements of O and A, respectively.

Definition 6. The symmetric-difference of two concepts (O_1, A_1) and (O_2, A_2), is the concept $D= (O_1 \underset{\bullet}{+} O_2, (A_1 \underset{\bullet}{+} A_2)^C)$, where $O_1 \underset{\bullet}{+} O_2$ and $A_1 \underset{\bullet}{+} A_2$ are the usual set-theoretic symmetric-differences of O_1 and O_2 or A_1 and A_2, respectively.

The set C of all concepts, with the two operations intersection ($\underset{\bullet}{\cap}$) and symmetric-difference D is proved to have the order of a lattice.

The symmetric-difference shows us the dissimilarities of the system of concepts and the intersection the similarities.

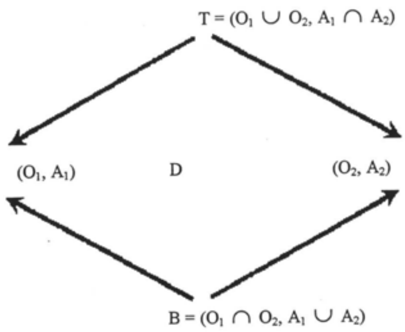


Figure 1.

Where D is the symmetric-difference of (O_1, A_1) and (O_2, A_2). The arrows give the subordinated concepts. (O_1, A_1), (O_2, A_2) and D are located on the same level and, consequently, there is no order among them.

Definition 7. We call distance d(X,Y) of two sets X and Y, the non-negative integer expressing the number of elements of the set $X \underset{\bullet}{+} Y$, that is of their symmetric-difference (in symbols $n(X \underset{\bullet}{+} Y)$). So, $d(X,Y) = n(X \underset{\bullet}{+} Y)$.

The three known properties of a distance hold:

1. $d(X,Y)= n(X \underset{\bullet}{+} Y) \geq 0$ and $d(X,X) = n(X \underset{\bullet}{+} X)= n(\Phi)=0$
2. $d(X,Y)= n(X \underset{\bullet}{+} Y) = n(X \underset{\bullet}{+} Y) = d(Y,X)$, since $X \underset{\bullet}{+} Y = Y \underset{\bullet}{+} X$
3. $d(X,Y) + d(Y,Z) = n(X \underset{\bullet}{+} Y) + n(Y \underset{\bullet}{+} Z)$.

We observe that $(X \underset{\bullet}{+} Y) \underset{\bullet}{+} (Y \underset{\bullet}{+} Z) = X \underset{\bullet}{+} Z$. Generally, $A \underset{\bullet}{+} B=(A \cup B) - (A \cap B)$, but

$n(A \underset{\bullet}{+} B) = [n(A) + n(B) - n(A \cap B)] - n(A \cap B) = n(A) + n(B) -2n(A \cap B)$.

So, $n(X \underset{\bullet}{+} Z) = n(X \underset{\bullet}{+} Y)+ n(Y \underset{\bullet}{+} Z) - 2n [(X \underset{\bullet}{+} Y) \cap (Y \underset{\bullet}{+} Z)] \Rightarrow n(X \underset{\bullet}{+} Y) + n(Y \underset{\bullet}{+} Z) \geq n(X \underset{\bullet}{+} Z)$. Consequently, the third property is valid.

Let's go, now, to the concepts. We can take $d(O_1\ O_2)=n\ (O_1 \underset{\bullet}{+} O_2)$, which is a distance between objects, but it does not say many things, since it is quantitative but not qualitative: two sets of objects may have many different elements, coming from the same homogenous population (Statistics ....). Besides, we are not working with objects or attributes, but with both of them, that is concepts. The symmetric-difference $O_1 \underset{\bullet}{+} O_2$ of the objects, has the icon $(A_1 \underset{\bullet}{+} A_2)^c$. So, if we want the real distance of O_1 and O_2, we must check $(A_1 \underset{\bullet}{+} A_2)^c$.

$d(A_1,A_2)=n(A_1 \underset{\bullet}{+} A_2)=n(\Omega') - n((A_1 \underset{\bullet}{+} A_2)^c)$, where Ω' is the set of all attributes (in our certain application). So, $n((A_1 \underset{\bullet}{+} A_2)^{\circ})=n(\Omega') - d(A_1,A_2)$. n(Ω') is a constant. Consequently, if the distance of the attributes is increasing, $n((A_1 \underset{\bullet}{+} A_2)^c)$ is decreasing and the distance of the objects is, accordingly, decreasing. The explanation comes naturally: if we have a large range of attributes, this range can fit only to a small range of objects.

2 General Theory of Terminology

The proposed algebraic structure is in complete accordance with the General Theory of Terminology. It gives, also, acceptable results from the view of cognition and learning.

All the situations, “real” or “imaginary” can be expressed. Let's take an isolated object O^1. As soon as we discover an isolated attribute a^1 we have the concept (O^1, a^1). With a second isolated attribute a^2 we have:

$$(O^1, a^1) \dot{\cup} (O^1, a^2) = (O^1 \cup O^1, a^1 \cap a^2) = (O^1, a^1 \cap a^2) = (O^1, \varnothing)$$

(because we suppose that a^1 and a^2 are different attributes) and

$$(O^1, a^1) \underset{\bullet}{\cap} (O^1, a^2) = (O^1 \cap O^1, a^1 \cup a^2) = (O^1, a^1 \cup a^2).$$

So, we have created a new concept: $(O^1, a^1 \cup a^2)$. This means that the isolated object O^1 has both attributes a^1 and a^2.

With a third isolated attribute a^3 we take three new concepts, $(O^1, a^1 \cup a^3)$, $(O^1, a^2 \cup a^3)$ and $(O^1, a^1 \cup a^2 \cup a^3)$. We proceed, till we create the concept (O^1, A_1), where A_1 is the set of all the attributes of O^1.

So, for every object, we go to deeper levels of order as we discover more attributes of it. The meaning is that for every new attribute, the object is classified to an always thinner classification. It may seem strange that, though we add attributes, we go to subordinate concepts but, really this is an advantage and very logical: the more attributes we add, the more we specify the object, the closer we go to it.

On the other hand, there are concepts with no order among them. For example (O^1, a^1) and (O^1, a^2) are located in the same level: neither superordinated nor subordinated the one to the other, but, also, not equal.

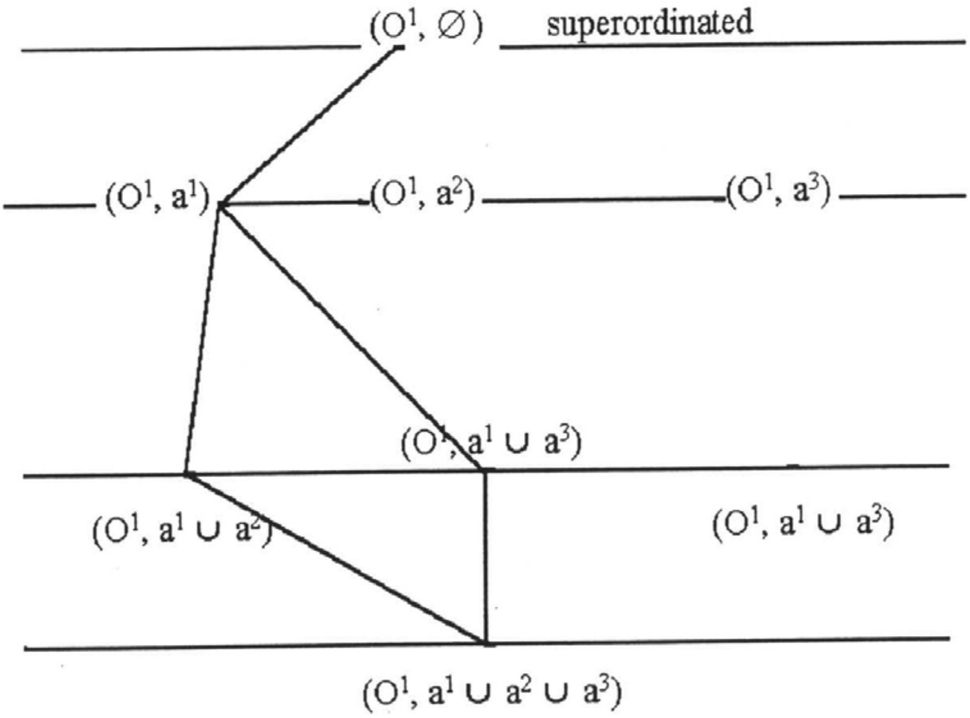


Figure 3.

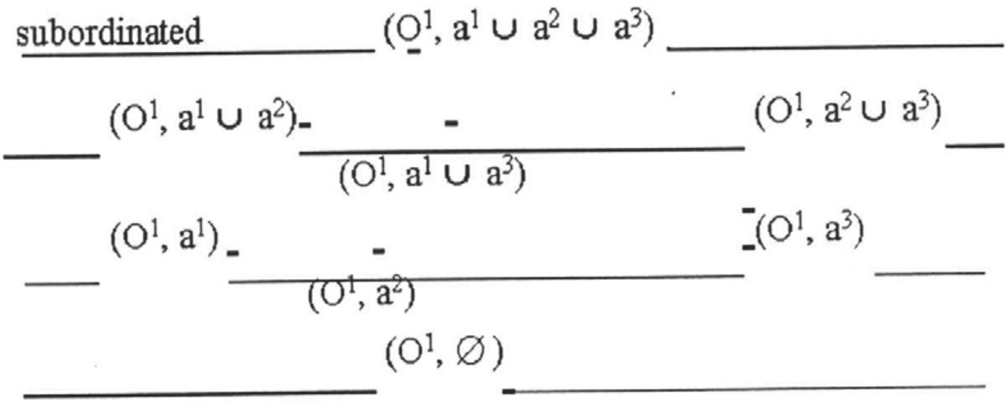


Figure 4.

Figure 4, if seen from bottom to top, gives the "natural" impression that the final concept (O^1, $a^1 \cup a^2 \cup a^3$) is built, step by step, from the empty set.

Suppose we have the concepts (O^1, A_1), (O^2, A_2) and (O^3, A_3), where O^1, O^2, O^3 are isolated objects and A_1, A_2, A_3 their sets of attributes, respectively. This means that every one of them stands at the top of a figure like Figure 4. We begin to take unions ($\cup$) and intersections ($\underset{\bullet}{\cap}$) between them and after-wards between them and the first results and so on.

From the unions we find: ($O^1 \cup O^2$, $A_1 \cap A_2$), ($O^1 \cup O^3$, $A_1 \cap A_3$), ($O^2 \cup O^3$, $A_2 \cap A_3$) and ($O^1 \cup O^2 \cup O^3$, $A_1 \cap A_2 \cap A_3$).

The intersections give us always the empty set in the left part of every couple because the isolated objects are supposed to be distinct. In this way, ($\varnothing$, A) means that the empty set has all the attributes of the set A. There is no problem to accept such a thing because, to the empty set, we may assign every attribute. Moreover, a concept like ($\varnothing$, A) can be useful in the structure because, e.g., ($\varnothing$, A) $\dot{\cup}$ (B, C) = ($\varnothing \cup$ B, A $\cap$ C) = (B, A $\cap$ C), which is a new result. The attributes A may correspond to an object not yet discovered (like the case of some planets, comets and atomic particles) or to an object that should be created or named (terminography work), so that we make an extension of our system.

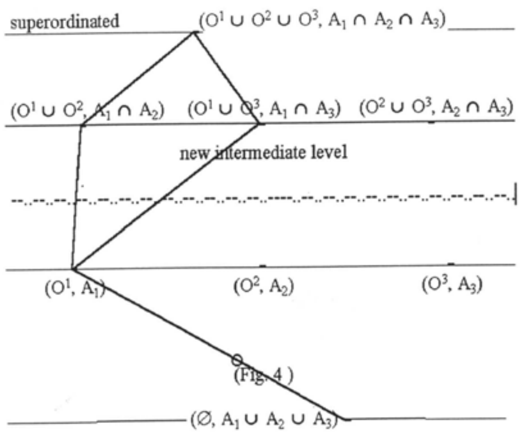


Figure 5.

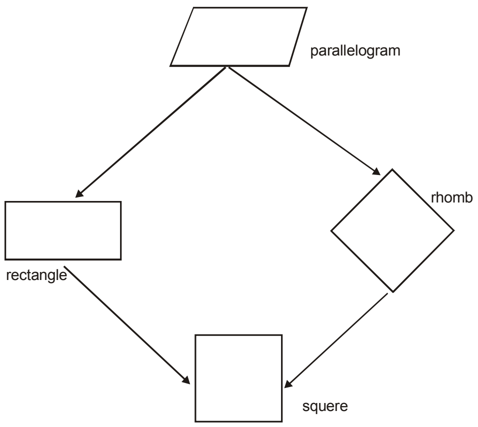
The cycle in Figure 5 represents the Figure 4.

Figure 4 gives us another possibility: all the concepts involving the same object O1, form another sublattice (class). So, we have all the possible scenarios concerning this object.
Figure 5 shows us the same result with Figure 3, but with a whole set A1 of attributes, in stead of the isolated attribute a1. This means that we can impose a whole set of conditions.
The access to information in not linear, but follows the structure of the lattice. Of course, real applications may give a sublattice of A, or, if we are unlucky, no order among the concepts.

In every case, if we, afterwards, create all the pζossible combinations (by using $\underset{\bullet}{\cap}$, $\dot{\cup}$ and the symmetric-differences), we find a complete lattice.
Links are a main characteristic of lattices. Conceptual lattices are created naturally from the attributes of their objects. For example, in Figure 3, from the top to the bottom there are two paths concerning the object O1 and the attribute a1. The smallest sublattice is the “elementary rhomb” of Figure 1.

3 Examples

a) from Geometry:



The structure is not a tree: a rectangle with equal sides becomes a square, but, also, a rhomb, with equal angles becomes a square. A square can be considered as a descendant of the rectangle, but also, of the rhomb.
The four concepts (parallelogram, rectangle, rhomb, square) form a lattice: {rectangle}

$\dot{\cup}$ {rhomb} = {parallelogram} and {rectangle} $\underset{\bullet}{\cap}$ {rhomb} = {square}.

b) from vehicles:

i) a submarine can move on the surface of the sea, but, also, under the surface (or rest on the bottom of the sea!). Is it a ship…?

ii) "Columbia" space vehicle, when coming back to the Earth, moves as a usual airplane…

iii) what about "Flying Dolphins", "Hovercrafts" and so on? They move on the surface of the sea (ships), but their movement obeys, partly, the laws of Physics obeyed by the airplanes!

So:

a ship has sails or/and propeller, moves only on the sea surface

a submarine has propeller, moves on the sea surface and under the surface

a Flying Dolphin has propeller, moves on the sea surface and slides with wings on it

wind-surfer has sail, slides on the sea surface.

It is obvious that the relations among these concepts do not form an hierarchy (a tree) E.g., one could say that a submarine is a special ship (that is, a ship capable of moving underwater), but the truth is that a submarine is something different than a ship (despite the fact that it can more, also, on the sea surface – as the ships do). Besides, there are ships moving with sails or with propeller and sails together (while the submarines use only propeller). Conclusion: objects and attributes are both necessary for a concept to be formed.

c) from Medicine:

i) "reading" an X-ray picture is a very difficult task. E.g., how can one discriminate pneumonia from cancer of lungs? To a certain extent, the two pictures are alike.

ii) fever and cough may lead to several different diseases…

iii) some substances can serve as a poison or a medicine (like women)! That's why, in Greece, we say "pharmako" = medicine and "pharmaki" = poison.

d) from every day life:

i) In German language, women use the expression "meine Man". But, it not clear what they mean by the term "Man": their husband, their boy-friend or something else…?

ii) Several newspapers (especially in Greece) use in their title the term "free"…

iii) Several political parties (all over the world) use in their title the term "democratic"… In the USA, the two major political parties are the "Democrats" and the "Republicans" (which means, also, democrats…)

iv) Almost all regimes claim that they have free elections…

v) A famous Greek painter has expressed the following opinion: "you are what you declare to be"! This is a great truth! According to the set of attributes you present each time, you belong to the corresponding class (=concept) of people…

vi) Virtual Reality is based, exactly, on the set of attributes we present each time. Women know something more about that, using make-up e.t.c.

vii) A recent research in Europe has revealed a strange fact: Tse Guevara is a symbol not only for many parties of the left political wing, but, also, for several parties of the right wing!!… It seems that a subset of attributes of the personality of Tse belongs to the path or semi-lattice or lattice or, generally, the network of the left way or thinking, while another subset belongs to the right way of thinking. Maybe, Tse does not belong neither to the one nor to the other wing (frame)…

viii) "It is so, if you consider it so" (Luigi Pirantello)

ix) Double-agents are a good example of belonging, partly, to two different (?) systems ("masters").

x) A few years ago, something terrible was discovered: the main lawyer supporting one political party, was, also, the main lawyer of the opponent political party!!! (…?)

4 Remarks

All lattices are not equivalent. Some of them are "richer" than the others. "Rich" means that they are consisted of more concepts, more possible situations.

Professor R. Wille takes as concepts only the standardized ones. Obviously, this fits well with robots, but not with human beings. Freedom and fantasy can give birth to many partly different or absolutely new objects-we do not have the right to exclude them. Our proposed system of concepts is not closed but open. Open to new concepts, open to probability, open to fuzziness, open to not predefined situations.

Our classification is not imposed from above (…), but it comes naturally from the attributes of the objects themselves. We have defined operations between concepts: union and intersection to express similarities (common objects or common attributes) and symmetric – differences between objects. So, we can make our classifications as thin as we want. Prof. Wille does not provide operations and, moreover, he does not treat differences. Obviously, another mentality … But, real life is full of differences and we are not allowed to normalize human thought!

We use couples in order to define concepts, because, in this way, we express the relativity of the objects. People are taught to think with trees – this is the simple and not annoying way…But, real life is full of complexities and lattice is one of the instruments to express them. In a tree, the child comes from only one parent (…), while in the lattice every child has two parents. Indeed, the child is the link between the two parents. Hierarchies are not the unique kind of order (…).

General conclusion: two concepts can be interrelated either by having common objects or by having common attributes.

5 Bibliography

ALTY and COMBS (1984): "Expert Systems", NCC Publitions.

BIRKHOFF, G. (1967): «Lattice Theory», third edition, Amer. Math. Soc., Providence. R.I., U.S.A.

FELBER H. (1984): "Terminology Manual", UNESCO, Paris.

KECHAJOPOULOU N. (1985): "Introduction to the Theory of Lattices", Athens.

SOTIROPOULOS M.T. (1989): «From Terminology to Informatics via Conceptual – Mathematical Models», «Terminologie et Traduction», No 3, 35-141.

SOTIROPOULOS M.T., «A conceptual-mathematical model for Data Bases and Expert – Systems», Doctor's Dissertation submitted to the School of Natural and Technical Sciences, Johannes Kepler University, Linz, Austria.

SOTIROPOULOS M.T. (1995): «A Mathematical Model of Concepts for Knowledge Extraction from Statistical Data», Proceedings of the Second International Conference of New Techniques and Technologies for Statistics (NTTS – 95), Bonn, Germany, 20 – 22 November, 465 – 471.

SOTIROPOULOS M.T. (1992): Mathematical Theory of Concepts: a mew tool for the application of Logo to Education», Poster Proceedings of the 12th World Computer Congress of the IFIP, Madrid, Spain, September 7 –11, 84.

TWAY, Linda E, (1992): «Work with Multimedia», Management Information Source, Inc., New York.

WILLE, r. (1988): «Restructuring Lattice Theory», Rival I., (ed): «Ordered Sets», Reidel Publ. Co.

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